EQUATION REDUCIBLE TO HOMOGENEOUS

1. $\dfrac{dy}{dx} = \dfrac{x+2y-3}{2x+y-3}$

Put $x = x'+h$ and $y = y'+k$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{dy'}{dx'}$

$\Rightarrow \dfrac{dy'}{dx'} = \dfrac{x'+h+2y'+2k-3}{2x'+2h+y'+k-3}$

$= \dfrac{x'+2y'+(h+2k-3)}{2x'+y'+(2h+k-3)}$

I have to find h and k so that

$h+2k = 3 \quad \times 2 \quad = 2h+4k = 6$

$2h+k = 3 \qquad\qquad 2h + k = 3$

$\Rightarrow 3k = 3 \Rightarrow k = 1$

$\Rightarrow 2h = 2 \Rightarrow h = 1$

$\Rightarrow \dfrac{dy'}{dx'} = \dfrac{x'+2y'}{2x'+y'}$

Put $y' = vx'$

$\Rightarrow \dfrac{dy'}{dx'} = v + x'\dfrac{dv}{dx'}$

$\Rightarrow v + x'\dfrac{dv}{dx'} = \dfrac{x'+2vx'}{2x'+vx'} = \dfrac{1+2v}{2+v}$

$\Rightarrow x'\dfrac{dv}{dx'} = \dfrac{1+2v}{2+v} - v = \dfrac{1+\cancel{2v}-\cancel{2v}-v^2}{v+2} = \dfrac{1-v^2}{2+v}$

$\Rightarrow \dfrac{2+v}{1-v^2} \cdot dv = \dfrac{dx'}{x'}$

$$\Rightarrow 2\int \frac{dv}{1-v^2} + \frac{(-1)}{2}\int \frac{-2v}{1-v^2}\,dv = \int \frac{dx'}{x'}$$

$$\Rightarrow \cancel{2}\cdot\frac{1}{\cancel{2}}\log\left|\frac{1+v}{1-v}\right| - \frac{1}{2}\log(1-v^2) = \log x' + c$$

$$\Rightarrow \log\left|\frac{1+\frac{y'}{x'}}{1-\frac{y'}{x'}}\right| - \frac{1}{2}\log\left(1-\frac{y'^2}{x'^2}\right) = \log x' + c$$

$$\Rightarrow \log\left|\frac{x'+y'}{x'-y'}\right| - \frac{1}{2}\log\left|\frac{x'^2-y'^2}{x'^2}\right| = \log x' + c$$

$$\Rightarrow \log\left|\frac{(x-1)+(y-1)}{(x-1)-(y-1)}\right| - \frac{1}{2}\log\left|\frac{(x-1)^2-(y-1)^2}{(x-1)^2}\right| = \log(x-1) + c$$

$$\Rightarrow \log\left|\frac{x+y-2}{x-y-}\right| - \frac{1}{2}\log\left\{(x-1)^2-(y-1)^2\right\} + \log(x-1) = \log(x-1) + c$$

$$\Rightarrow \log(x+y-2) - \log(x-y) - \frac{1}{2}\log\left\{(x-1)^2-(y-1)^2\right\} = c$$

$\Rightarrow$

$$\Rightarrow \log(x+y-2) - \log(x-y) - \frac{1}{2}\log(x+y-2) - \frac{1}{2}\log(x-y)$$

$$\Rightarrow \frac{1}{2}\log(x+y-2) - \frac{3}{2}\log(x-y) = c$$

$$\Rightarrow \log(x+y-2) - 3\log(x-y)^3 = 2c$$

$$\Rightarrow \log(x+y-2) - \log(x-y)^3 = -\log K$$

$$\Rightarrow \log(x+y-2) + \log K = \log(x-y)^3$$

$$\Rightarrow \log K(x+y-2) = \log(x-y)^3$$

$$\Rightarrow K(x+y-2) = (x-y)^3$$ Ans

2. $$\frac{dy}{dx} = \frac{6x-2y-7}{3x-y+4}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2(3x-y)-7}{3x-y+4}$$

Put $3x-y = z$

$$\Rightarrow 3 - \frac{dy}{dx} = \frac{dz}{dx}$$

$$\Rightarrow 3 - \frac{dz}{dx} = \frac{dy}{dx}$$

$$\Rightarrow 3 - \frac{dz}{dx} = \frac{2z-7}{z+4}$$

$$\Rightarrow 3 - \frac{2z-7}{z+4} = \frac{dz}{dx}$$

$$\Rightarrow \frac{3z+12-2z+7}{z+4} = \frac{dz}{dx}$$

$$\Rightarrow \frac{z+19}{z+4} = \frac{dz}{dx}$$

$$\Rightarrow \frac{z+4}{z+19} \cdot dz = dx$$

$$\Rightarrow \frac{(z+19-15)}{z+19} dz = dx$$

$$\Rightarrow \int dz - 15\int \frac{dz}{z+19} = \int dx$$

$$\Rightarrow z - 15\log(z+19) = x + c$$

$$\Rightarrow 3x - y - 15\log(3x-y+19) = x + c$$

$$\Rightarrow 2x - y - c = 15\log(3x-y+19)$$ Ans

3. $\frac{dy}{dx} = \frac{4x+6y+5}{3y+2x+4}$

$\Rightarrow \frac{dy}{dx} = \frac{2(3y+2x)+5}{3y+2x+4}$

Put $3y+2x = z$

$\Rightarrow 3\frac{dy}{dx} + 2 = \frac{dz}{dx}$

$\Rightarrow 3\frac{dy}{dx} = \frac{dz}{dx} - 2$

$\Rightarrow \frac{dy}{dx} = \frac{1}{3}\left(\frac{dz}{dx} - 2\right)$

$\Rightarrow \frac{1}{3}\left(\frac{dz}{dx} - 2\right) = \frac{2z+5}{z+4}$

$\Rightarrow \frac{dz}{dx} - 2 = \frac{6z+15}{z+4}$

$\Rightarrow \frac{dz}{dx} = \frac{6z+15+2z+8}{z+4} = \frac{8z+23}{z+4}$

$\Rightarrow \frac{z+4}{8z+23}\,dz = dx$

$\Rightarrow \frac{1}{8}\int \frac{(8z+32)}{8z+23}\,dz = \int dx$

$\Rightarrow \frac{1}{8}\int \frac{(8z+23+9)}{8z+23}\,dz = \int dx$

$$= \frac{1}{8}\int dz + \frac{19}{8}\int \frac{dz}{8z+23} = \int dx$$

$$= \frac{1}{8}\left[\int dz + \frac{9}{8}\int \frac{8\,dz}{8z+23}\right] + k = x$$

$$= \frac{1}{8}\left[z + \frac{9}{8}\log(8z+23)\right] + k = x$$

$$= 3y + 2x + \frac{9}{8}\log\{8(3y+2x)+23\} + k = 8x$$

$$\Rightarrow \quad 6x - 3y = \frac{9}{8}\log\{8(2x+3y)+23\} + k$$

$$\Rightarrow \quad 2x - y = \frac{3}{8}\log\{8(2x+3y)+23\} + k \quad A$$

$(6x - 5y + 1)dy = (2x - y + 1)dx$

$\Rightarrow \frac{dy}{dx} = \frac{2x - y + 1}{6x - 5y + 1}$

Put $x = x' + h$, $y = y' + K$

$\Rightarrow \frac{dy}{dx} = \frac{dy'}{dx'}$

$\Rightarrow \frac{dy'}{dx'} = \frac{2x' + 2h - y' - K + 1}{6x' + 6h - 5y' - 5K + 1}$

$\Rightarrow \frac{dy'}{dx'} = \frac{2x' - y' + (2h - K + 1)}{6x' - 5y' + (6h - 5K + 1)}$

Now, I have to find h and K so that

$2h - K + 1 = 0 \quad \times 3$

$6h - 5K + 1 = 0$

$6h - 3K + 3 = 0$ } by subtracting

$-2K - 2 = 0 \Rightarrow -2(K+1) = 0 \Rightarrow K = -1$

Again, $2h = -1 - 1 = -2$

$\Rightarrow h = -1$

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By adding, $2h-1=0 \Rightarrow h=\frac{1}{2}$, $K=\frac{1}{2}$

$h-k=0 \Rightarrow h=K \Rightarrow$

Now, $\frac{dy'}{dx'} = \frac{x'+y'}{x'-y'}$

Put $y' = vx'$

$\Rightarrow \frac{dy'}{dx'} = v + x'\frac{dv}{dx'}$

$\Rightarrow v + x'\frac{dv}{dx'} = \frac{x'+vx'}{x'-vx'} = \frac{1+v}{1-v}$

$\Rightarrow x'\frac{dv}{dx'} = \frac{1+v}{1-v} - v = \frac{1+\cancel{v}-\cancel{v}+v^2}{1-v} = \frac{1+v^2}{1-v}$

$\Rightarrow \frac{1-v}{1+v^2}\,dv = \frac{dx'}{x'}$

$\Rightarrow \int\frac{dv}{1+v^2} - \frac{1}{2}\int\frac{2v\,dv}{1+v^2} = \int\frac{dx'}{x'}$

$\Rightarrow \tan^{-1}v - \frac{1}{2}\log(1+v^2) = \log x' - \log c$

$\Rightarrow \cancel{\tan^{-1}\left(\frac{y'}{x'}\right) - \tan^{-1}}$

$\Rightarrow \tan^{-1}\left(\frac{y'}{x'}\right) - \frac{1}{2}\log\left(1+\frac{y'^2}{x'^2}\right) = \log x' - \log c$

$\Rightarrow \tan^{-1}\left(\frac{y-K}{x-h}\right) - \frac{1}{2}\log\left(\frac{x'^2+y'}{x'^2}\right) = \log x' - \log c$

$\Rightarrow \tan^{-1}\left(\frac{y-\frac{1}{2}}{x-\frac{1}{2}}\right) - \frac{1}{2}\log\left\{\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{2}\right)^2\right\} + \log x' - \log c$

$+\log c$

$$3\cancel{\log x'} + \log(y' - x') = 3\cancel{\log x'} + \log c^3 + 4\log(5y' - 2x')$$

$$\log\{y + 1 - x + 1) = \log c^3 + \log\{5(y+1) - 2(x+1)\}^4$$

$$\log(y - x + 2) = \log c^3 + \log(5y - 2x - 3)^4$$

$$\log(y - x + 2) = \log c^3(5y - 2x - 3)^4 \quad \text{Ans//}$$

$$\Rightarrow \frac{dy'}{dx'} = \frac{2x'-y'}{6x'-5y'}$$

Put $y' = vx'$

$$\Rightarrow \frac{dy'}{dx'} = v + x'\frac{dv}{dx'}$$

$$\Rightarrow v + x'\frac{dv}{dx'} = \frac{2x'-vx'}{6x'-5vx'} = \frac{2-v}{6-5v}$$

$$\Rightarrow x'\frac{dv}{dx'} = \frac{2-v}{6-5v} - v = \frac{2-v-6v+5v^2}{6-5v}$$

$$\Rightarrow x'\frac{dv}{dx'} = \frac{5v^2-7v+2}{6-5v}$$

$$\Rightarrow \frac{6-5v}{5v^2-7v+2} \cdot dv = \frac{dx'}{x'}$$

$$\Rightarrow \frac{6-5v}{(5v-2)(v-1)} \cdot dv = \frac{dx}{x'}$$

$$\Rightarrow \frac{-20}{3}\int \frac{dv}{5v-2} + \frac{1}{3}\int \frac{dv}{v-1} = \int \frac{dx}{x'}$$

$$\Rightarrow \frac{-4}{3}\int \frac{5\,dv}{5v-2} + \frac{1}{3}\int \frac{dv}{v-1} = \int \frac{dx}{x'}$$

$$\Rightarrow \frac{-4}{3}\log(5v-2) + \frac{1}{3}\log(v-1) = \log x' + \log c$$

$$\Rightarrow \frac{-4}{3}\log\left(\frac{5y'}{x'}-2\right) + \frac{1}{3}\log\left(\frac{y'}{x'}-1\right) = \log x' + \log c$$

$$\Rightarrow -4\log\left(\frac{5y'-2x'}{x'}\right) + \log\left(\frac{y'-x'}{x'}\right) = 3\log x' + 3\log c$$

$$\Rightarrow -4\log(5y'-2x') + 4\log x' + \log(y'-x') - \log x' = 3\log x' + 3\log c$$

Type - II

13. $(x+y+1)dx = (2x+2y+3)dy$

$$\Rightarrow \frac{dy}{dx} = \frac{x+y+1}{2(x+y)+3}$$

Put $x+y = z$

$$\Rightarrow 1+\frac{dy}{dx} = \frac{dz}{dx} \qquad \Rightarrow \frac{dy}{dx} = \frac{dz}{dx} - 1$$

$$\Rightarrow \frac{dz}{dx} - 1 = \frac{z+1}{2z+3}$$

$$\Rightarrow \frac{dz}{dx} = \frac{z+1}{2z+3} + 1 = \frac{z+1+2z+3}{2z+3} = \frac{3z+4}{2z+3}$$

$$\Rightarrow \frac{2z+3}{3z+4} \cdot dz = dx$$

$$\Rightarrow \frac{\frac{2}{3}\left(3z+\frac{9}{2}\right)}{3z+4} \cdot dz = dx$$

$$\Rightarrow \frac{2}{3}\int \frac{\left(3z+4+\frac{1}{2}\right)}{3z+4} dz = dx$$

$$\Rightarrow \frac{2}{3}\int \frac{(3z+4)}{3z+4} dz + \frac{1}{3}\int \frac{dz}{3z+4} = \int dx$$

$$\Rightarrow \frac{2}{3}\int dz + \frac{1}{9}\int \frac{3\,dz}{3z+4} = \int dx$$

$$\Rightarrow \frac{2z}{3} + \frac{1}{9}\log(3z+4) = x + c$$

$$\Rightarrow 6z + \log(3z+4) = 9x + 9c$$

$$\Rightarrow 6(x+y) + \log(3x+3y+4) = 9x + 9c$$

$$\Rightarrow \log(3x+3y+4) = 3x - 6y + 9c$$ Ans

14. $(x+y+1)dx = (2x+2y+1)dy$

$$\Rightarrow x+\cdot\frac{dy}{dx} = \frac{x+y+1}{2(x+y)+1}$$

Put $x+y=z$

$$\Rightarrow 1+\frac{dy}{dx} = \frac{dz}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{dz}{dx} - 1$$

$$\Rightarrow \frac{dz}{dx} - 1 = \frac{z+1}{2z+1}$$

$$\Rightarrow \frac{dz}{dx} = \frac{z+1}{2z+1} + 1$$

$$\Rightarrow \frac{dz}{dx} = \frac{z+1+2z+1}{2z+1} = \frac{3z+2}{2z+1}$$

$$\Rightarrow \frac{2z+1}{3z+2}\cdot dz = dx$$

$$\Rightarrow \frac{2\cdot\left(3z+\frac{3}{2}\right)}{3\cdot\ 3z+2}\cdot dz = dx$$

$2\left(z+\frac{1}{2}\right)$

$$\Rightarrow \frac{2}{3}\left(\frac{3z+2-\frac{1}{2}}{3z+2}\right)dz = dx$$

$2-\frac{3}{2}$

$\frac{1}{2}$

$$\Rightarrow \frac{2}{3}\int\frac{(3z+2)}{3z+2}dz - \frac{1}{3}\int\frac{dz}{3z+2} = \int dx$$

$$\Rightarrow \frac{2}{3}\int dz - \frac{1}{9}\int\frac{3\,dz}{3z+2} = \int dx$$

$$\Rightarrow \frac{2z}{3} - \frac{1}{9}\log(3z+2) - \ldots$$

$\Rightarrow 6z - \log(3z+2) = 9x + 9c$

$\Rightarrow 6(x+y) - \log\{3(x+y)+2\} = 9x + 9c$

$\Rightarrow 6x + 6y - \log(3x+3y+2) = 9x + 9c$

$\Rightarrow 6y - 3x + 9c = \log(3x+3y+2)$

$\Rightarrow 6y - 3x + K = \log(3x+3y+2)$ Ans

15. $(x+y+1)\,dy = (2x+2y-1)\,dx$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{2x+2y-1}{x+y+1} = \dfrac{2(x+y)-1}{x+y+1}$

Put $x+y = z$ $\Rightarrow 1 + \dfrac{dy}{dx} = \dfrac{dz}{dx}$

$\Rightarrow \dfrac{dz}{dx} - 1 = \dfrac{2z-1}{z+1}$

$\Rightarrow \dfrac{dz}{dx} = \dfrac{2z-1}{z+1} + 1 = \dfrac{2z-\cancel{1}+z+\cancel{1}}{z+1} = \dfrac{3z}{z+1}$

$\Rightarrow \dfrac{z+1}{3z} \cdot dz = dx$

$\Rightarrow \dfrac{1}{3}\int dz + \dfrac{1}{3}\int \dfrac{dz}{z} = \int dx$

$\Rightarrow \dfrac{1}{3} z + \dfrac{1}{3} \log z = x + c$

$\Rightarrow \dfrac{z}{3} + \log z = 3x + 3c$

$\Rightarrow x + y + \log(x+y) = 3x + 3c$

$\Rightarrow 2x - y - 3c = \log(x+y)$ Ans

Now, $\int \frac{v+1}{1+v^2} \cdot dv = -\int \frac{dx}{x}$

$$\Rightarrow \frac{1}{2}\int \frac{2v}{1+v^2} dv + \int \frac{dv}{1+v^2} = -\int \frac{dx}{x'}$$

$$\Rightarrow \cancel{\frac{1}{2}} \log(1+v^2) + \tan^{-1} v = -\log x'$$

$$\Rightarrow \log\left(1+\frac{y'^2}{x'^2}\right) + \tan^{-1}\left(\frac{y'}{x'}\right) = -\log x'$$

$$\Rightarrow \log(x'^2+y'^2) - \cancel{\log x'} + \tan^{-1}\left(\frac{y'}{x'}\right) = -\cancel{\log x'} - \log c$$

$$\Rightarrow \tan^{-1}\left(\frac{y-k}{x+h}\right) + \log\{(x-h)^2+(y-k)^2\} + \log c = 0$$

$$\Rightarrow \tan^{-1}\left(\frac{y-1}{x-2}\right) + \log c\{(x-2)^2+(y-1)^2\} = 0 \text{ Ans}$$

2. $(x-y)\,dy = (x+y+1)\,dx$.

$$\Rightarrow \frac{dy}{dx} = \frac{x+y-1}{x-y}$$

Put $x = x'+h$ and $y = y'+k$

$$\Rightarrow \frac{dy}{dx} = \frac{dy'}{dx'}$$

$$\Rightarrow \frac{dy'}{dx'} = \frac{x'+h+y'+K-1}{x'+h-y'-K} = \frac{(x'+y')+(h+K-1)}{(x'-y')+(h-K)}$$

Now we have to find h and K so that,

$h+K-1 = 0$

$h-K = 0$

16. $(2x+3y-5)\,dy + (2x+3y-1)\,dx = 0$

$\Rightarrow (2x+3y-5)\,dy = -(2x+3y-1)\,dx$

$$\Rightarrow \frac{dy}{dx} = -\frac{(2x+3y-1)}{2x+3y-5}$$

Put $2x+3y = z$

$$\Rightarrow 2+3\cdot\frac{dy}{dx} = \frac{dz}{dx} \qquad \Rightarrow 3\cdot\frac{dy}{dx} = \frac{dz}{dx} - 2$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{3}\left(\frac{dz}{dx} - 2\right)$$

$$\Rightarrow \frac{1}{3}\left(\frac{dz}{dx} - 2\right) = \frac{1-z}{z-5}$$

$$\Rightarrow \frac{dz}{dx} - 2 = \frac{-3z+3}{z-5}$$

$$\Rightarrow \frac{dz}{dx} = \frac{-3z+3}{z-5} + 2 = \frac{-3z+3+2z-10}{z-5} = \frac{-z-7}{z-5}$$

$$\Rightarrow \frac{z-5}{z+7}\cdot dz = -dx$$

$$\Rightarrow \frac{z+7-12}{z+7}\,dz = -dx$$

$$\Rightarrow \int dz - 12\int\frac{dz}{z+7} = -\int dx$$

$\Rightarrow z - 12\log(z+7) = -x + c$

$\Rightarrow 2x+3y - 12\log(2x+3y+7) = -x + c$

$\Rightarrow 3x+3y - 12\log(2x+3y+7) = c$

$\Rightarrow 3x+3y = 12\log(2x+3y+7) + 4k$

$\Rightarrow x+y = 4\log(2x+3y+7) + k$ Ans